

or

$$\frac{\delta}{x} = 3.93 (Pr)^{-1/2} \left(Pr + \frac{20}{21} \right)^{1/4} \left(\frac{g \beta \theta_m}{\nu^2} \right)^{-1/4} x^{-3/4}$$

or

$$\frac{\delta}{x} = 3.93 (Pr)^{-1/2} \left(Pr + \frac{20}{21} \right)^{1/4} \left(\frac{g \beta \theta_m x^3}{\nu^2} \right)^{-1/4}$$

Introducing $Gr_x = \text{Grashof number} \equiv \frac{g \beta \theta_m x^3}{\nu^2}$ we get

$$\frac{\delta}{x} = 3.93 (Pr)^{-1/2} \left(Pr + \frac{20}{21} \right)^{1/4} Gr_x^{-1/4}$$

or

$$\frac{\delta}{x} = 3.93 \left[\frac{Pr + \frac{20}{21}}{Pr^2 Gr_x} \right]^{1/4} \quad (7.26)$$

Equation (7.26) gives the variation of boundary layer thickness along the height of the plate. Here thermal boundary layer thickness and hydrodynamic boundary layer thickness are assumed to be same.

7.2.1 Heat transfer coefficient

The heat flux from the wall may be expressed as

$$q'' = -k \left. \frac{\partial T}{\partial y} \right|_{y=0} = h_x (T_w - T_\infty)$$

The local heat transfer coefficient at any x - location is

$$h_x = \frac{-k(\partial T/\partial y)_{y=0}}{T_w - T_\infty}$$

or

$$h_x = -\frac{k}{(T_w - T_\infty)} \frac{\partial}{\partial y} \left[T_\infty + (T_w - T_\infty) \left(1 - \frac{y}{\delta} \right)^2 \right]_{y=0} = \frac{2k}{\delta}$$

or

$$\frac{h_x x}{k} = \frac{2x}{\delta} = \frac{2}{3.93} \left[\frac{Pr^2 Gr_x}{Pr + \frac{20}{21}} \right]^{1/4}$$

The local Nusselt number is given by

$$Nu_x = 0.508 \left[\frac{Pr^2 Gr_x}{(Pr + \frac{20}{21})} \right]^{1/4} \quad (7.27)$$

The average Nusselt number over the plate is given by

$$\overline{Nu}_L = \frac{1}{L} \int_0^L Nu_x dx = \frac{4}{7} Nu_L$$

where L is the length of the plate.

Note: In a field where free convection and forced convection both are present, free convection is negligible if $Gr/Re^2 \ll 1$, forced convection is negligible if $Gr/Re^2 \gg 1$. For a situation with combined free and forced convection, the condition is, $Gr/Re^2 \sim 1$.

We can also write

$$\frac{g\beta\theta_m L^3}{\nu^2} \sim \frac{U^2 L^2}{\nu^2}$$

An equivalent free convection velocity may be defined as

$$u \sim \sqrt{g\beta\theta_m L}$$

The above Eq. (7.27) is used for laminar boundary layers. It is customary to correlate the transition to turbulence if the Rayleigh number crosses a certain limit.

$$Ra_{x,c} = Gr_{x,c} Pr = \frac{g\beta(T_w - T_\infty)x^3}{\nu^2} \frac{\nu}{\alpha}$$

Flow becomes turbulent if $Ra_{x,c} \geq 10^9$

Some important correlations:

- Vertical plate: $\overline{Nu}_L = 0.68 + \frac{0.67 Ra_L^{1/4}}{[1 + (0.492/Pr)^{9/16}]^{4/9}}$, $10^{-1} < Ra_L < 10^9$.
- Horizontal plate: $L = \frac{A_s}{P} = \frac{\text{Surface area}}{\text{Perimeter}}$. If upper side of the plate is heated, then $\overline{Nu}_L = 0.54 Ra_L^{1/4}$, $10^4 \leq Ra_L \leq 10^7$.
- Long horizontal cylinder (due to Churchill and Chu [3]) $\overline{Nu}_D = \left[0.60 + \frac{0.387(Ra_D)^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}} \right]^2$, $10^{-5} \leq Ra_D \leq 10^{12}$.

7.3 VERTICAL CYLINDER

For the fluids, having Prandtl numbers of 0.7 and higher, a vertical cylinder may be treated as a vertical flat plate [2] when

$$\frac{D}{L} \geq \frac{35}{(Gr_L)^{1/4}}$$

Basically, if the thermal boundary layer is significantly smaller than the radius of the cylinder, the vertical flat plate and vertical cylinder have considerable geometrical similarity.

Figure 7.2 shows a plot of the ratio of the average Nusselt number for a vertical cylinder to that of a vertical plate as a function of the parameter $\xi = \frac{2\sqrt{2}}{(Gr_L)^{1/4}} \cdot \left(\frac{L}{R}\right)$. Here, R is the radius of the cylinder and L is the length.

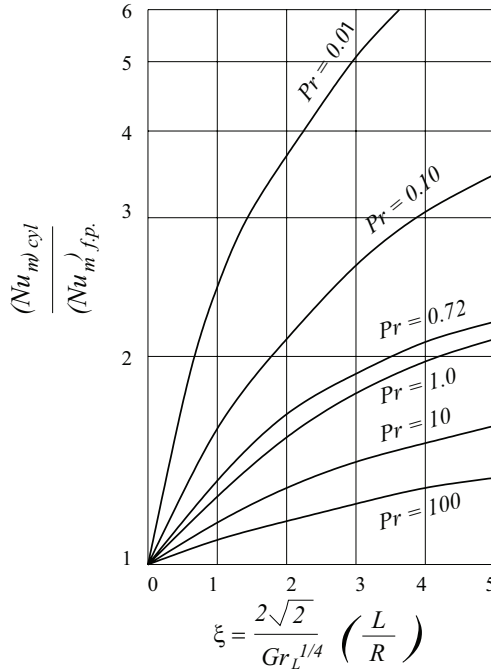


Figure 7.2 The ratio of the Nusselt number for a vertical plate to that of a vertical cylinder (courtesy of Ozisik [4])

7.4 PLUMES

A plume is the buoyancy induced flow resulting in a fluid, when energy is supplied continuously at just one location in the fluid. In practice free plumes, like free jets are generally turbulent. An axisymmetric plume is found to be laminar only if the Rayleigh number based on its heat source(q'') and height of the plume(H) is less than 10^{10} . Rayleigh number is given by

$$Ra_H = \frac{g\beta H^4 q''}{\alpha \nu k}$$

7.4.1 Integral Analysis of Steady Plume

The governing equations for the flow in a plume are the continuity equation, the Navier-Stokes equations, and the energy equation. Assuming the mean flows in a

7.5 MIXED CONVECTION IN RECTANGULAR CHANNEL

Let us consider flow of air or gases through a horizontal channel as shown in Fig. 7.4. Two important non-dimensional groups may be introduced here. One is Archimedes number, Ar and the other is Froude number, F_r . The reference temperature is T_∞ and the reference pressure is P_∞ .

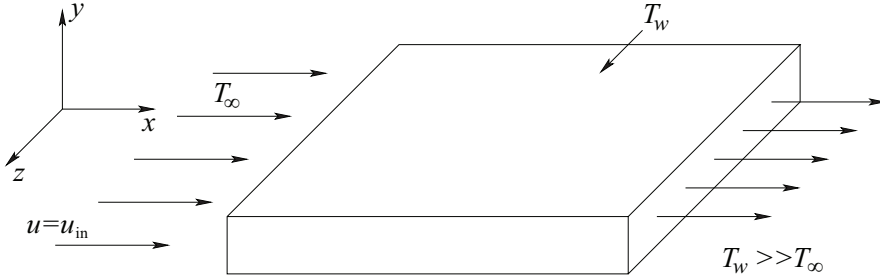


Figure 7.4 Interaction of buoyancy induced flow and forced flow through horizontal channel

The Froude number is given by, $F_r = \frac{V}{\sqrt{g\ell}}$ and Archimedes number is given by $Ar = \frac{\beta(T_w - T_\infty)}{F_r^2} = \beta(T_w - T_\infty)g\ell/V^2$. In the above mentioned definitions, V is the velocity and ℓ is the length scale. The Archimedes number is linked to Grashof number in the following way

$$Gr = Ar \times Re^2 = \frac{\beta(T_w - T_\infty)g\ell}{V^2} \frac{V^2 \ell^2}{\nu^2}$$

$$Gr = \frac{g\beta(T_w - T_\infty)\ell^3}{\nu^2}$$

Figure 7.4 shows combined buoyancy induced flow and forced convection flow through a horizontal channel. The following are the governing equations

Continuity:

$$\nabla \cdot \mathbf{V} = 0$$

x mom:

$$\frac{\partial u}{\partial t} + (\mathbf{V} \cdot \nabla)u = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla^2 u$$

y mom:

$$\frac{\partial v}{\partial t} + (\mathbf{V} \cdot \nabla)v = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \nabla^2 v + g\beta(T - T_\infty)$$

z mom:

$$\frac{\partial w}{\partial t} + (\mathbf{V} \cdot \nabla)w = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \nabla^2 w$$

Energy:

$$\frac{\partial T}{\partial t} + (\mathbf{V} \cdot \nabla)T = \alpha [\nabla^2 T]$$

After non-dimensionalization,

$$u^* = u/u_{in}$$

$$v^* = v/u_{in}$$

$$w^* = \frac{w}{u_{in}}$$

$$x^* = \frac{x}{L}$$

$$y^* = y/L$$

$$z^* = z/L$$

$$p^* = p/\rho u_{in}^2$$

$$\theta = (T - T_\infty)/(T_w - T_\infty)$$

$$t^* = t/(L/u_{in})$$

$$Re = \frac{u_{in} L}{\nu}$$

$$Gr = g\beta(T_w - T_\infty)L^3/\nu^2$$

$$Pr = \nu/\alpha$$

we get

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0 \quad (7.41)$$

$$\frac{Du^*}{Dt^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{Re} \nabla^2 u^* \quad (7.42)$$

$$\frac{Dv^*}{Dt^*} = -\frac{\partial p^*}{\partial y^*} + \frac{1}{Re} \nabla^2 v^* + \frac{Gr}{Re^2} \theta \quad (7.43)$$

$$\frac{Dw^*}{Dt^*} = -\frac{\partial p^*}{\partial z^*} + \frac{1}{Re} \nabla^2 w^* \quad (7.44)$$

$$\frac{D\theta}{Dt} = \frac{1}{Re \cdot Pr} \nabla^2 \theta \quad (7.45)$$

Gr/Re^2 is called Richardson number = R_i .

The problem can be solved numerically using coupled (Navier-Stokes and energy equations) solution technique applied to Eqs. (7.41) to (7.45). Many useful applications of a similar flow configuration have been discussed by Incropera et al. [6].

7.6 MIXED CONVECTION INCLUDING BUOYANCY AIDED AND BUOYANCY OPPOSED FLOWS

Let us consider the temperature conditions as depicted in Fig. 7.5

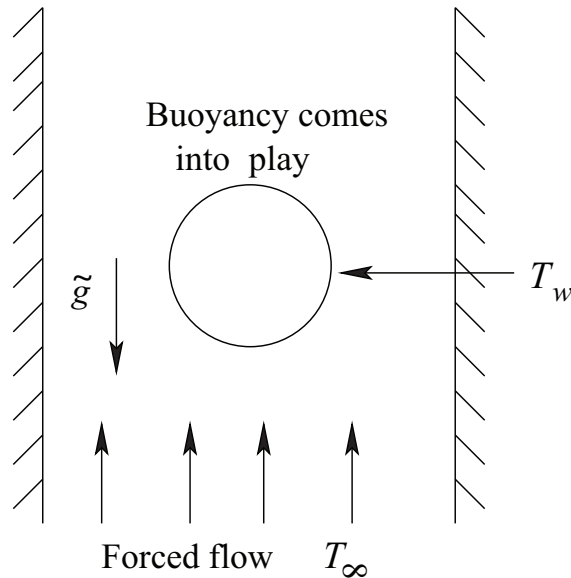


Figure 7.5 Mixed convection over a cylinder through parallel plate

$$T_w > T_\infty, Ri + ve$$

$$T_w < T_\infty, Ri - ve$$

In the mixed convection problem shown in Fig. 7.5 vortex shedding is found to stop at a critical Richardson number. The forced flow Reynolds number may be considered as 100. The following explanations are interesting (Singh et al. [7]).

The physical significance of negative Ri is culmination of a buoyancy force in a direction opposite to the flow direction. In the viscous region aft of the cylinder, the inertia force is opposed by the force due to an adverse pressure gradient, the negative buoyancy force, and the viscous force. Depending on the magnitude of Ri , the point of separation moves toward the leading edge, causing an early separation. This also broadens the wake. Vortex shedding becomes prominent in the entire regime of negative Ri . The shedding phenomenon continues to characterize the flow even at $Ri = 0$ and 0.1 (Fig. 7.6). For $Ri = 0.15$ (streamlines are not shown here) and above, the flow structure becomes different. The aiding buoyancy changes the dynamics altogether. Near the vicinity of the cylinder, even aft of the cylinder, the buoyancy force is added up with the inertia force in the face of combined pressure force and viscous force. The phenomenon results in a separation delay, and vortex shedding is also stopped. This observation is in agreement with earlier study by Chang and Sa [8]. Figure 7.6 shows isotherms for various Ri . The buoyancy opposed and buoyancy aiding cases are discerned in this figure. It can readily be

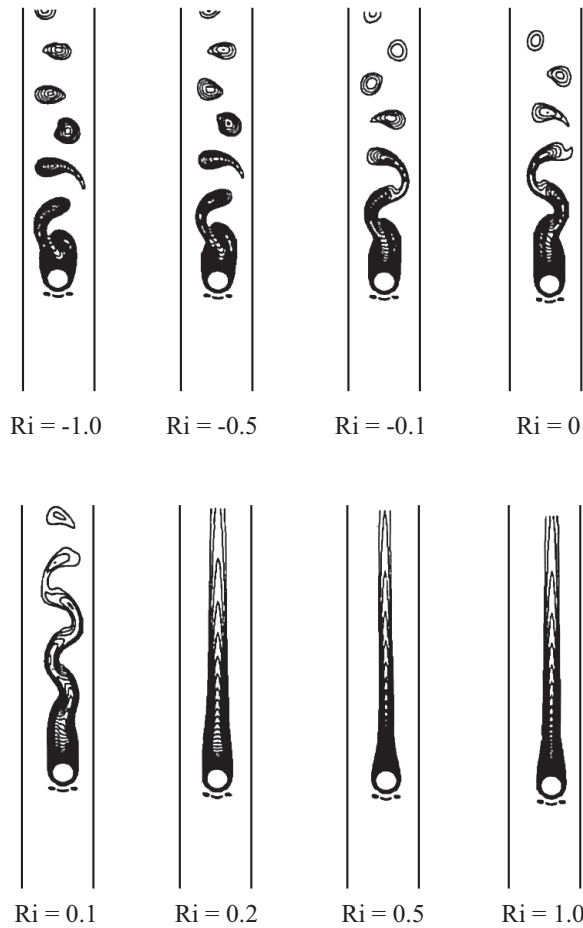


Figure 7.6 Isotherms for different Richardson numbers at $Re = 100$ and $D/H = 0.25$.

appreciated that the waviness of the isotherms downstream of the cylinder is greater for the buoyancy-opposed convection than for buoyancy-aiding convection. Beyond $Ri = 0.2$, the plume spread becomes narrow, and waviness disappears.

Concluding Remarks

Free convection has wide range of applications in cooling of electronic devices and computers. Some heat-generating electronic components are cooled using a fan or a pump too. Therefore, a decision has to emerge whether only free convection or mixed convection cooling is to be used for cooling of electronic components. The choice depends on the maximum allowable operating temperature of the equipment. The convection heat transfer rate from a surface at temperature T_w in a medium at T_∞ is given by

$$Q_{conv} = hA_w(T_w - T_\infty)$$

where h is the heat transfer coefficient and A_w is the surface area. Therefore, the surface temperature will be higher if h is low for the equipment. Free convection heat transfer coefficient is usually low.

Free convection or natural convection is a preferred mode of heat transfer. Usually blowers or pumps are not needed for free convection and as a consequence the problems, such as vibration, noise, etc. can be avoided. Free convection is usually appropriate for cooling low-power devices. It is very effective for the extended surfaces too. For the high-power devices, blowers or pumps are needed to maintain the operating temperature below the allowable limit. For even higher power devices, in order to maintain the surface temperature below the prescribed limit, phase-change cooling is utilized. Heat pipes are such cooling devices.

The rates of heat transfer in free or natural convection can be made much higher by operating the devices in the near-critical or supercritical regions of fluids. In such regimes, small changes in temperature or pressure yield large changes in the fluid thermophysical properties of the fluid. Significant change in the thermophysical properties of the fluid can bring about appreciable changes in the rate of convective heat transfer.

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EXERCISES

1. (a) What is Boussinesq approximation for the free convective flows?